Geometric And Variational Analysis.
In memory of Jan Malý.

9th June-15th June 2024, Będlewo, Poland

Short talks (in alphabetic order)

Milos Arsenovic (Belgrade University)

Restricted and non-restricted maximal functions in $\mathbb{D}^n$ with applications

Restricted and non restricted maximal functions acting on separately (α, β) harmonic functions in the unit polydisc are studied. Weak (1, 1) type estimates and norm estimates are given as well estimates in a Zygmund class for these maximal functions. These estimates are used to derive a. e. convergence results at the distinguished boundary of the unit polydisc. A convergence result at a single point of the distinguished boundary is also obtained in the class of bounded separately (α, β) harmonic functions. The above results can be viewed as generalizations of classical results on boundary behavior of n-harmonic functions in the unit disc, as well as more recent results on the boundary behavior of (α, β) harmonic functions in the unit disc.

Anna Balci (Bielefeld University and Charles University)

Fractals and large singular sets: In memory of Jan Malý

We present a general procedure to construct examples of convex scalar variational problems which admit minimizers with large singular sets. The dimension of the set of singularities is maximal and the minimizer has no higher integrability property (failure of Meyers property).This talk is based on join works with Lars Diening and Mikhail Surnachev.

Maciej Borodzik (University of Warsaw and Institute of Mathematics of Polish Academy of Sciences)

On equivariant Almgren theorem

The theorem of Almgren relates the homology groups of a manifold with homotopy groups of the space of integral chains on it. Given a manifold with a group action, and an orthogonal, finite-dimensional representation V of a finite group G, we construct a space of equivariant currents Z_V, whose equivariant homotopy groups compute Bredon homology of the manifold. This is a join project with Sławek Kolasiński and Wojciech Politarczyk.

Anisa Chorwadwala (Indian Institute of Science Education and Research (IISER) Pune)

Sharp bounds for higher Steklov-Dirichlet Eigenvalue

We consider a mixed Steklov-Dirichlet eigenvalue problem on smooth bounded domains in Riemannian manifolds. Under certain symmetry assumptions on multi connected domains in a Euclidean space with a spherical hole, we obtain isoperimetric inequalities for k-th Steklov-Dirichlet eigenvalues for k between 2 and $n + 1$. We extend Theorem 3.1 of [GPPS] from Euclidean domains to domains in space forms, that is, we obtain sharp lower and upper bounds of the first Steklov-Dirichlet eigenvalue on bounded star-shaped domains in the unit n-sphere and in the hyperbolic space.

[GPPS] Nunzia Gavitone, Gloria Paoli, Gianpaolo Piscitelli, and Rossano Sannipoli. An isoperimetric inequality for the first Steklov–Dirichlet laplacian eigenvalue of convex sets with a spherical hole. *Pacific Journal of Mathematics*, 320 (2) 241– 259, 2023.

Arthur Danielyan (University of South Florida)
On the boundary properties of bounded analytic functions

We present new results on the boundary values of analytic functions. Let Δ and T be the open unit disc and circle in the complex plane, respectively. By Fatou's theorem, a bounded analytic in Δ function f has radial (even angular) limits a.e. on T. Except the radial and angular limits of the function f, we also consider the unrestricted limits of f.

Theorem 1. *Let E be a subset on T. There exists a bounded analytic function in Δ which has no radial limit at the points of E, but has unrestricted limits at the points of $T \setminus E$ if and only if E is an F_σ set of Lebesgue measure zero.*

The necessity part of Theorem 1 is trivial. The condition that E an F_σ is obvious. The condition that E is of Lebesgue measure zero is obvious by Fatou's theorem. In fact, Theorem 1 is a converse to Fatou's theorem. Also, the sufficiency part of Theorem 1 strengthens a known theorem of Lohwater and Piranian (1957).

The talk will present also the solution to a problem on the disc algebra functions proposed by M. von Renteln (1980) and published in a well-known open problem list in complex analysis.

Miguel García Bravo (University Complutense of Madrid)
About the boundaries of Sobolev extension domains

Given a domain $\Omega \subset \mathbb{R}^n$ we say that Ω is a $W^{1,p}$-extension domain if there exists a constant $C > 0$ so that for every Sobolev function $f \in W^{1,p}(\Omega)$ there exists $F \in W^{1,p}(\mathbb{R}^n)$ so that $F|_\Omega = f$ and $\|F\|_{W^{1,p}(\mathbb{R}^n)} \leq C\|f\|_{W^{1,p}(\Omega)}$. The question whether a domain has or does not have the Sobolev extension property has been widely studied during the last sixty years. In general the extension is possible whenever the domain has nice geometric properties, like having a Lipschitz boundary or being uniform (these results are due to Calderón, Stein and Jones).

This talk has two main objectives. Both of them are related to get a better understanding of "how big" (in the sense of measure) the boundaries of Sobolev extension domains can be. Even though it is known that $W^{1,p}$-extension domains for $1 \leq p < \infty$ satisfy that $\mathcal{L}^n(\partial\Omega) = 0$, in general it could happen that $\dim_{\mathcal{H}}(\partial\Omega) = n$. Still, there are two approaches to meaningfully study this question. One of them is trying to limit the topology of our domain Ω and the other one is trying to study only those points of $\partial\Omega$ that play a more important role for the extension (for us these are points where the boundary self-intersects). We present two new results:

- While for Sobolev extension domains $\Omega \subset \mathbb{R}^2$ that are homeomorphic to the unit disk we must have $\dim_{\mathcal{H}}(\partial\Omega) < 2$, we show that there exists a domain $\Omega \subset \mathbb{R}^3$ homeomorphic to the unit ball that is a $W^{1,p}$-extension domain for every $1 \leq p \leq \infty$ but $\dim_{\mathcal{H}}(\partial\Omega) = 3$.
- By inspecting the possibility that the boundary of Sobolev extension domains may self-intersect, we show some Hausdorff dimensional estimates on this so-called set of *two-sided points.*

This is a joint work with Tapio Rajala and Jyrki Takanen.

Amiran Gogatishvili (Institute of Mathematics of the Czech Academy of Sciences)
Orlicz-Sobolev spaces

We will give several definitions of fractional order Orlicz-Sobolev spaces. More attention will paid to the upper gradient characterization and decomposition methods in the framework of Fourier analysis. It will be considered also the quasi-normed cases.

Anatoly Golberg (Holon Institute of Technology)
Absolute continuity in higher dimensions

In 1999, Jan Malý contributed a classical counterpart of absolute continuity in the sense of Banach or in the sense of Tonelli to $\mathbb{R}^n$. This notion has cast light on various problems including the area, coarea and degree formulas. Note that the n-absolute continuity forms a proper subclass of Sobolev class $W^{1,n}$ including validity of Lusin's (N)-property. Further investigations, mainly due to Csörnyei, Hencl and Bongiorno, essentially extended and weakened the assumptions in Malý's ideas, and provided a wide spectrum of absolute continuity/bounded variation concepts. Such main results are discussed in our talk.

We also intend to illustrate and connect these results for the classes of finitely bi-Lipschitz mappings and mappings of finite metric and area distortion, whose definitions rely on a metric approach and further can be easily extended to a more general setting than $\mathbb{R}^n$. Some our recent results and illustrating examples are presented in the talk as well.

Luigi Greco (Università degli Studi di Napoli "Federico II")
Noncoercive parabolic obstacle problems

We prove existence results for Cauchy–Dirichlet problem for convection–diffusion parabolic equations with singular coefficients in the convective term. Related obstacle problem is also solved.

Our operator is not coercive, the obstacle function is time–dependent, irregular and the coefficients in the lower order term belong to a borderline mixed Lebesgue–Marcinkiewicz space.

The results are contained in some joint works with Fernando Farroni, Gioconda Moscariello and Gabriella Zecca.

Pankaj Jain (South Asian University)
TBA

David Kalaj (University of Montenegro)
Gaussian curvature conjecture for minimal graphs

In this talk we present a recent solution of longstanding Gaussian curvature conjecture of a minimal graph S over the unit disk ([1]). This conjecture states the following. For any minimal graph lying above the entire unit disk, the Gaussian curvature at the point above the origin satisfies the sharp inequality $|K| < \pi^2/2$. The conjecture is first reduced to the estimation of the Gaussian curvature of certain Scherk type minimal surfaces over some bicentric quadrilaterals inscribed in the unit disk containing the origin. Then we make a sharp estimate of the Gaussian curvature of those minimal surfaces over those bicentric quadrilaterals at the point above the zero. Our proof uses complex-analytic methods since minimal surfaces that we consider allow conformal harmonic parametrization.

[1] D. Kalaj, P. Melentijević, *Gaussian curvature conjecture for minimal graphs.*
https://doi.org/10.48550/arXiv.2111.14687

Bernd Kirchheim (University of Leipzig)
On (some) pointwise PDEs

The work goes back to a question by S. Konjagin (2018) about solutions to $u_x = u_y$ without any further assumptions.

We worked on it with Jan Malý and later found, that this question was already asked and partial solved by R. Baire and others. Related simple PDEs are discussed as well.

Aleksis Koski (Aalto University)
Homeomorphic extensions with controlled Hölder-continuity

The Kirszbraun extension theorem tells us that a Lipschitz map on any subset of $\mathbb{R}^n$ may be extended into a Lipschitz map of the whole space. Meanwhile, the Jordan-Schönflies theorem says that an embedding of the unit circle into a Jordan curve of the plane may be extended as a homeomorphism of the whole plane to itself. In this talk, we ask whether one can construct extensions which preserve both the properties of regularity and injectivity in such an extension. We will cover some recent results on homeomorphic extensions with sharp Hölder-continuity estimates. Based on a joint work with Stanislav Hencl.

Stefan Krömer (ÚTIA AV ČR Prague)
Measure structured deformations

Structured deformations were introduced by Del Piero and Owen as a model for deformations of solids exhibiting additional submacroscopic disarrangements like microscopically grained slips. Macroscopically, they are described by a deformation map which is piecewise differentiable with possible jumps (or, more generally, of class SBV), together with an additional internal variable in L^1 which can be used to keep track of submacroscopic disarrangements. We discuss the natural (weak*) closure of this class in $BV \times \mathcal{M}$ (functions of bounded variation and measures) and the relaxation of associated variational problems, extending earlier results of Choksi and Fonseca.

Joint work: Martin Kružík (ÚTIA AV ČR Prague), Marco Morandotti (Polytechnic Turin), Elvira Zappale (Sapienza University of Rome)

Michał Łasica (Institute of Mathematics of the Polish Academy of Sciences)
Jump discontinuities of minimizers in variational denoising

We consider a class of variational regularization models such as the Rudin-Osher-Fatemi image denoising model. The models involve minimization of a convex functional composed of a data fidelity term and a regularizing term. Since the latter is typically of linear growth in the gradient, the minimizers are BV functions which may have jump discontinuities, corresponding to sharp contours in images.

Under a mild regularity assumption on the regularizer, we show that the jump set of a minimizer is contained in the jump set of the datum, provided that the latter is a BV function. For general bounded measurable data, we show that the jump set of a minimizer is a subset of a generalized jump set of the datum, which is contained in the complement of its Lebesgue points. We also give estimates on the magnitude of the jumps.

This is joint work with Antonin Chambolle.

Lenka Slavíková (Charles University)
Almost-compact embeddings

Almost-compact embeddings between Banach function spaces are a useful tool for proving compactness of Sobolev embeddings. In this talk, I will review some known results about almost-compact embeddings and I will also explain how attending a course taught by Jan Malý influenced my research on this topic.

Filip Soudský (Technical University of Liberec)
Gagliardo–Nirenberg inequality via a new point-wise estimate

We prove a new type of pointwise estimate of the Kałamajska–Mazya–Shaposhnikova type, where sparse averaging operators replace the maximal operator. It allows us to extend the Gagliardo–Nirenberg interpolation inequality to all rearrangement invariant Banach function spaces without any assumptions on their upper Boyd index, i. e. omitting problems caused by unboundedness of maximal operator on spaces close to L^1. In particular, we remove unnecessary assumptions from the Gagliardo–Nirenberg inequality in the setting of Orlicz and Lorentz spaces. The applied method is new in this context and maybe seen as a kind of sparse domination technique fitted to the context of rearrangement invariant Banach function spaces.

T. V. Anoop (IIT Madras)

On reverse Faber-Krahn inequalities first eigenvalue of the p-Laplacian

In 1961, Payne-Weinberger showed that '*among the class of membranes with a given area A, free along the interior boundaries and fixed along the outer boundary of a given length L_0, the concentric annulus has the highest fundamental frequency.*' We discuss the extension of this result to the higher dimension ($N \geq 3$) for the p-Laplacian, namely, the reverse Faber-Krahn inequalities for the first eigenvalue of the p-Laplace operator on domains with holes satisfying the Dirichlet boundary condition on the outer boundary and the Neumann boundary conditions on the inner boundaries.

This is joint work with Dr. Ashok Kumar (srasoku@gmail.com) and Dr. Mrityunjoy Ghosh (ghoshmrityunjoy22@gmail.com).

References

[1] T. V. Anoop, and Ashok Kumar, K, On reverse Faber-Krahn inequalities, *Journal of Mathematical Analysis and Applications*, **485** (2020), 537-568.

[2] T. V. Anoop, and M. Ghosh, Reverse Faber-Krahn inequalities for Zaremba problems, *arXiv:2205.12717* to appear in Topological Methods in Nonlinear Analysis

Vesna Todorcevic (Mathematical Institute of the Serbian Academy of Sciences and Arts)

H^p-theory for quasiregular mappings

We consider some possible extensions of a classical result of Zinsmeister to quasiregular mappings. A particular emphasis will be given to recent results.

Matti Vuorinen (University of Turku)

Visual angle metric in the half plane

For a given domain $G \subset \mathbb{R}^2$ and $a, b \in G$, the visual angle metric is defined as follows

$$v_G(a,b) = \sup\{\measuredangle(a,z,b) : z \in \partial G\}\,.$$

We only consider the cases $G \in \{\mathbb{B}^2, \mathbb{H}^2\}$.

In [1] an explicit formula for the visual angle metric involving the hyperbolic metric was given for the unit disk $\mathbb{B}^2$. It seems natural to expect that finding a similar formula for the upper half plane $\mathbb{H}^2$ should be possible. The fundamental difficulty here is that the hyperbolic metric is invariant under Möbius automorphisms of $\mathbb{H}^2$ while the visual angle metric is not. Moreover, we were not able to extend the proof of [1] to the upper half plane case. This talk reports on join research [2], where an explicit formula for $v_{\mathbb{H}}(a,b)$ was found in terms of the hyperbolic metric.

We apply this result to prove a Hölder continuity result in the $v_{\mathbb{H}}$ metric for K-quasiregular mappings $f : \mathbb{H}^2 \to \mathbb{H}^2$. The result is sharp for Möbius transformations.

The proof utilizes computer algebra methods combined with manual postprocessing.

References

[1] M. Fujimura, R. Kargar, and M. Vuorinen, Formulas for the visual angle metric, J. Geom. Anal. (to appear) arXiv:2304.04485, 14pp.

[2] M. Fujimura, O. Rainio, and M. Vuorinen, The visual angle metric of the half plane, arXiv:2404.08942, 33pp.

Zheng Zhu (Beihang University)

The pointwise inequality for Sobolev functions on outward cuspidal domains

In this talk, we will show that every Sobolev function has Hasjlasz-type pointwise inequality on arbitrary outward cuspidal domains. This generalizes a result by Ramanov. Joint with Eriksson-Bique, Koskela and Maly.